ON THE IMPORTANCE OF PRECESSION FOR THE BADMINTON SHUTTLECOCK

STUDY OF ANGULAR MOMENTUM

PURPOSE

The purpose of this project is **to study the influence of angular movements on the trajectory** of a badminton shuttlecock.

Sport	$2R$ (cm)	M (g)	U_{max} (m/s)	U_∞ (m/s)	$Re = 2RU_\infty/\nu$	C_D	$R\Omega_0$ (m/s)	F_D/Mg	F_L/Mg
Badminton	6.0	5.00	137	6.7	3×10^4	0.64	0.019	420	0.1
Tennis	6.5	55.0	73.0	22	1×10^5	0.56	10	11	3
Ping-pong	4.0	2.50	32.0	10	3×10^4	0.36	17	10	18
Squash	4.0	24.0	78.0	34	1×10^5	0.30	3.1	5.4	0.8
Jai alai	6.5	120	83.0	41	2×10^5	0.38	0.20	4.1	0.03
Golf	4.2	45.0	91.0	48	2×10^5	0.23	7.7	3.6	1.4
Volleyball	21	210	37.0	20	4×10^5	0.25	5.3	3.4	2.1
Soccer	21	450	51.0	30	5×10^5	0.24	9.1	2.9	2.3
Softball	9.7	190	47.0	33	3×10^5	0.38	6.1	2.0	0.7
Baseball	7.0	145	54.0	40	2×10^5	0.38	6.6	1.8	0.6
Cricket	7.2	160	53.0	40	3×10^5	0.40	3.4	1.8	0.3
Lacrosse	6.3	143	50.0	48	3×10^5	0.35	0.20	1.1	0.01
Handball	19	450	27.0	36	6×10^5	0.20	0.60	0.56	0.07
Basketball	24	650	16.0	31	6×10^5	0.24	0.75	0.27	0.06

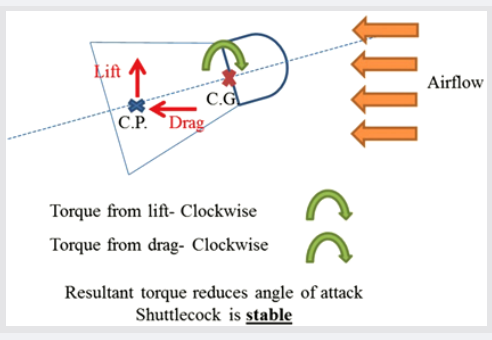
MOTIVATION

The speeds reached by shuttlecocks after impact are the highest measured of any sports projectile. The generated trajectories are very peculiar and different from those of any other sports projectile.

If we study the forces and moments applied to the shuttlecock, we can **better understand its flight**.

AERODYNAMIC CHARACTERISTICS

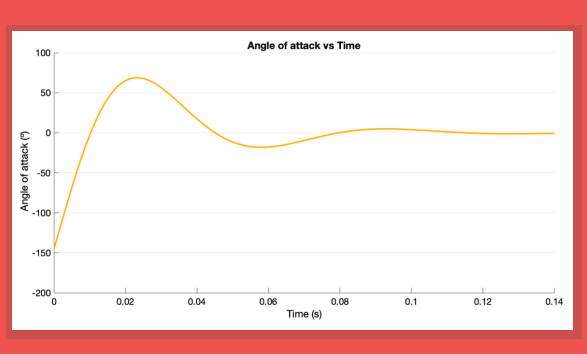
- Aerodynamic forces are decisive in the complex trajectory of the projectile.
- Historically, the internal design has always been the same; a heavy cork with several lightweight feathers attached to it.
- Due to this mass distribution, the Center of Gravity (C.G.) is located in front of the Center of Pressure (C.P.). This difference of positions means that the aerodynamic forces always stabilise the shuttlecock.

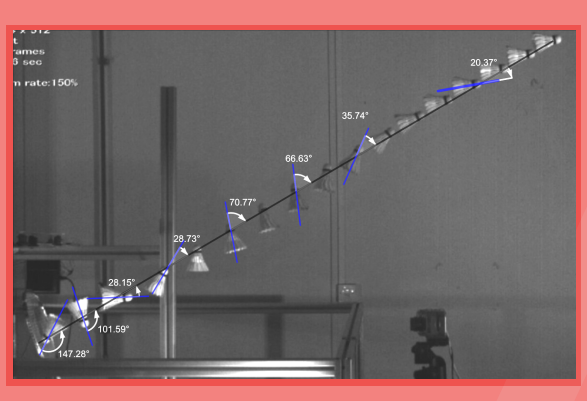


-ANGULAR MOMENTUM IN A TYPICAL SHUTTLECOCK TRAJECTORY-

1 FIRST PART OF THE TRAJECTORY. PITCH MOMENT

- The pitch rotation occurs in the non-stationary part of the trajectory, just after the impact.
- This angular movement, when restoring, will always tend to align the axis of the shuttlecock (blue line) with the velocity vector (black line), meaning that it will tend to decrease the angle of attack α until it is canceled.
- Average stabilization time = 0.15 s.



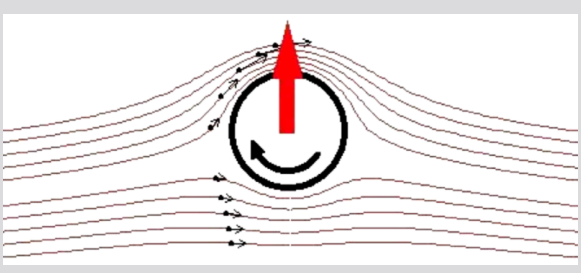


2 SECOND PART OF THE TRAJECTORY. PRECESSIONAL MOVEMENT

- The experience shows that the trajectory of a shuttlecock is not purely two-dimensional. The shuttlecock moves a small but appreciable distance on the axis perpendicular to the plane of the trajectory.
- The shuttlecock, in its movement through the air, in addition to rotating on its pitch axis, does so on its longitudinal axis. This rotation is called spin and is responsible of the generation of a gyroscopic movement with its respective precessions and nutations of the shuttlecock.
- This gyroscopic movement can be produced by two different effects, the **Magnus effect** and the **gyroscopic effect**.

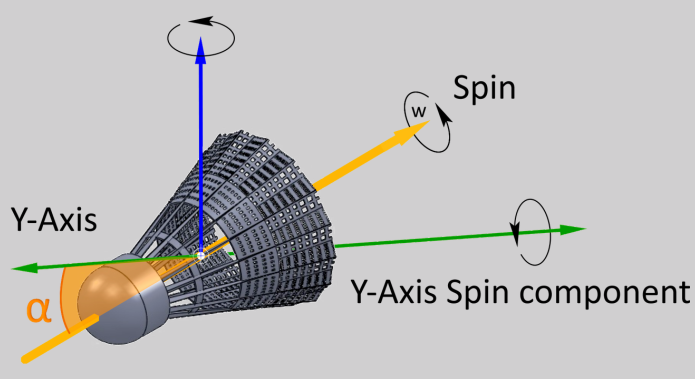
MAGNUS EFFECT

The **Magnus effect** occurs when a body moves while immersed in a fluid and at the same time has a rotation on itself. The body departs from its straight path because of pressure differences that develop in the fluid as a result of velocity changes (Bernoulli's principle) induced by the spinning body.



The component of spin outside the velocity vector (Y-Axis) will be the only one that can produce a precession movement and move the shuttlecock away from the 2D plane of the trajectory.

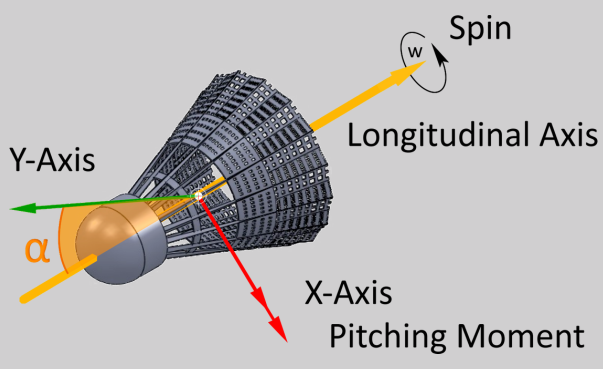
Spin component outside Y-Axis



GYROSCOPIC EFFECT

The **gyroscopic effect** is produced at the instant when a rotating body with respect to an axis OO' has a torque applied on a different axis OO''. This causes a movement of the axis of rotation of the body, which begins to rotate with an angular speed.

With the shuttlecock, whose axis of spin rotation is its longitudinal axis, there is a moment applied in pitch due to changes in the angle of attack α, so we can say it satisfies the gyroscopic conditions.

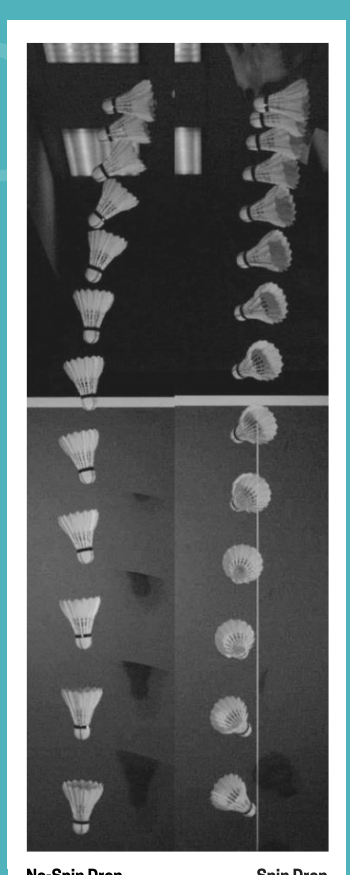


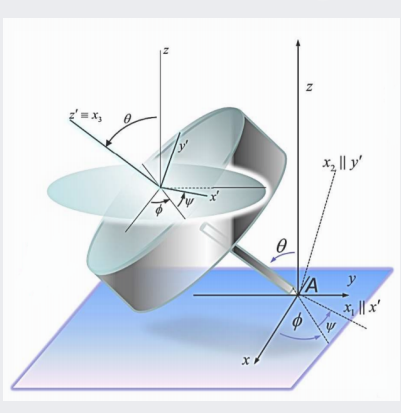
SECONDARY PRECESSION

The combination between spin and pitch moment causes the shuttlecock to start precessing with a variation of the yaw angle, as can be seen in the spin drop.

In addition, the variation of the yaw angle produces a restoring moment, which forces the shuttlecock to return to its original yaw direction. This change in yaw angle, along with the spin, produces a pitch moment that now affects the angle of attack. The spin drop (right) shows how the shuttlecock head resists falling, in contrast to the no-spin drop (left). This is the **secondary precession**.

This interchange of gyroscopic processes in yaw and pitch angle makes the shuttlecock move in a spiral motion, like a corkscrew.





HYPOTHESIS: AXISYMMETRIC TOP MODEL

We assume that the shuttlecock moves with respect to the velocity vector like a axisymmetric spinning top, keeping the CG as a fixed point. **Euler equations** and **Lagrangian Mechanics** will be used to model the movement.

RESULTS

- The **total deviation** in a typical clear stroke **is 0.17 meters** (Figure 1).
- Due to the precession and secondary precession, the concept of **triangularity** can be observed, which implies that these effects cause the shuttlecock **to descend more abruptly** during its fall, which is characteristic of the feather shuttlecocks (Figure 2).

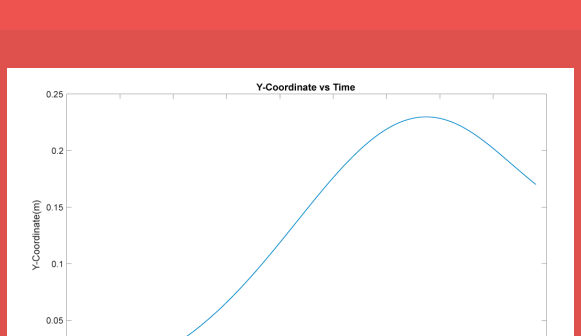


Fig. 1 - Shuttlecock deviation vs Time. Typical Clear stroke.

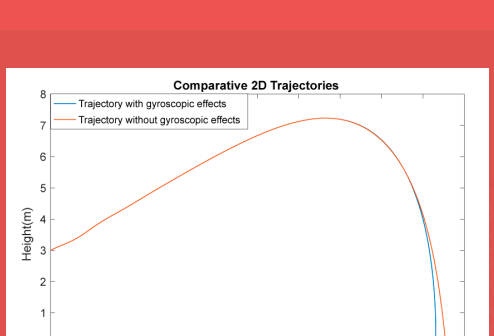


Fig. 2 - Comparative 2D Trajectories. Typical Clear stroke.

- The **2D trajectories simulations** historically made **can be** completed and **extended to 3D** with this model (Figure 3).

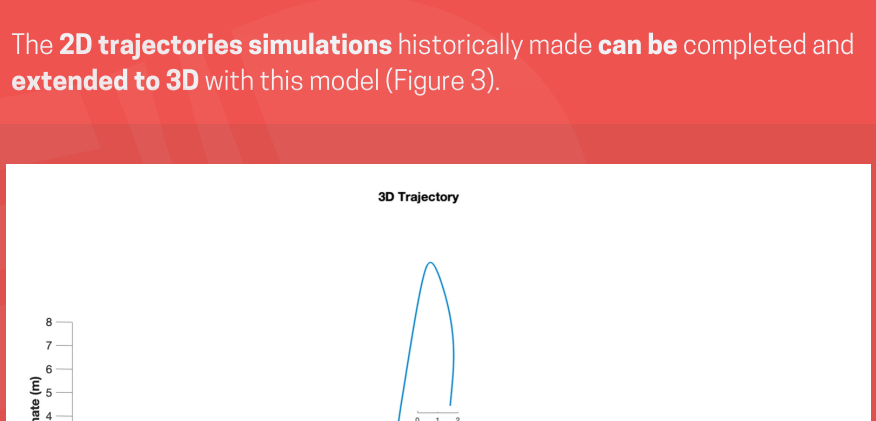


Fig. 3 - 3D Trajectory. Typical Clear stroke.

References

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[2] Calvin, Lin, and JH Yeo. *Badminton shuttlecock stability: Modelling and simulating the angular response of the turnover*. J Sports Engineering and Technology 1–10, 2015.

[3] Cohen C, Darbois-Texier B. *The aerodynamic wall*, 2014. Proc. R. Soc A 470:20130497.

[4] Calvin, Phd. *Understanding and evaluation of badminton shuttlecocks through flight dynamics and experimental approach*. Nanyang Technological University.

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